**A Little Background on our Productive Struggle and Persistence Exercises**

**Learning Opportunities and Rich Problem Cycle Phases**

(from Givvin and Saunders, forthcoming)

*Note: The Carnegie Community College work involves two developmental math courses: Quantway and Statway, or more generally referred to as Pathway instruction (i.e., pathways through to college credit bearing coursework).*

**Learning Opportunities**

Explicit Connections: By explicit attention to connections, we mean that connections among mathematical/statistical facts, procedures, and ideas should be addressed in an explicit and public way.

This could include discussing the mathematical meaning underlying procedures, asking questions about how different solution strategies are similar to and different from each other, considering the ways in which mathematical problems build on each other or are special (or general) cases of each other, attending to the relationships among mathematical ideas, and reminding students about the main point of the lesson and how this point fits within the current sequence of lessons and ideas. (Hiebert & Grouws, 2007, p. 383)

A review of findings from across multiple studies—some teacher-centered, others student-centered—suggest that teaching for conceptual understanding leads to improvement not only in conceptual understanding but also in procedural skill. The reverse has not found to be true (Hiebert & Grouws, 2007). Thus, when we suggest that the focus of Pathway instruction is on concepts, we are not suggesting that knowledge of procedures is unimportant, but rather that instruction focused on concepts is the better way to achieve both learning outcomes.

Productive Struggle: Struggling with problems – both large and small – is a core part of the Pathway instructional experience. We take our definition of struggle from the research literature on mathematics education.

We use the word *struggle* to mean that students expend effort to make sense of mathematics, to figure something out that is not immediately apparent. We do *not* use *struggle* to mean needless frustration or extreme levels of challenge created by nonsensical or overly difficult problems. We do not mean the feelings of despair that some students can experience when little of the material makes sense. The struggle we have in mind comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehendible but not yet well formed. By struggling with important mathematics we mean the opposite of simply being presented information to be memorized or being asked only to practice what has been demonstrated. (Hiebert & Grouws, 2007, pp. 387-388)

This idea that struggle facilitates conceptual understanding is consistent with what we know from cognitive psychology. Sometimes called “desirable difficulties” (Bjork, 1994; Bjork & Bjork, 2011; Schmidt & Bjork, 1992) the general idea is that certain kinds of difficulties that can be introduced into the learning process actually improve learning—especially deeper learning as measured on delayed post-tests—despite learners’ strong perceptions that they don’t. When students struggle, they work more effortfully to make sense of the situation, which in turn leads to interpretations more connected to what they already know. Similar themes run through the work of Dewey (1910), Festinger (1957), and Vygotsky (1978).

Deliberate Practice: The literature suggests that repeating a behavior over and over is not an effective method of reaching maximal levels of performance. Pashler (2008) writes that “most current mathematics texts mass practice problems relating to a given topic into one problem set presented immediately following textual presentation of that topic. Our data suggest that—at least for promoting retention—this may be a grievous error” (p. 189). Research shows further that performance is best increased as a result of deliberate, spaced efforts aimed at improvement. Deliberate practice consists of tasks that are invented to overcome gaps in understanding, apply what is learned, and deepen understanding and facility with key concepts. These activities are highly structured and are designed to improve performance and strengthen understanding; performance on them is carefully monitored to provide cues for ways to improve it further. Deliberate practice requires effort and individuals are motivated to practice because practice improves performance (Ericcson et al., 1993, 2008). For these reasons, the Pathways are not characterized by long series of similar problems, but rather by carefully chosen questions that guide students to a deeper understanding of concepts.

**Problem Cycle**

The learning opportunities are a central element of the instructional design principles for the Statway and Quantway. The *problem cycle* is another element of the design principals. Regarded as the mechanism by which the learning opportunities would largely be achieved, the problem cycle is modeled after the typical organization of mathematics lessons in Japan (Shimizu, 1999), where instruction is often organized around four phases. As part of the development of the larger Carnegie Framework for Improving Teaching (FIT) each of the four phases were defined in terms of their purpose to the larger problem cycle. That work was completed by instructors and researchers who have been working on Statway and Quantway since its inception.

Problem Launch: The purpose of the launch is to prepare students for *productive struggle* -- to create a shared understanding of the problem to be worked on, make clear why solving it is important, and stimulate a variety of ways to think about the problem.

Working the Problem: The purpose of the working phase is to engage students in *productive struggle* with the problem and the concepts and to study students’ ways of thinking to prepare for the discussion. The purpose of this phase is NOT to ensure that all students get the correct answers.

Discussing the Problem: The purpose of discussing the problem is to make public students’ ways of thinking (correct and incorrect), encourage students to learn new ways of thinking by understanding each other, and *connect their thinking to the key concept(s)*.

Conclusion: The purpose of the conclusion is to concisely highlight the key concepts drawn from students’ thinking, express the concepts with appropriate notation and representations, *and explicitly connect the lesson concept(s)* with the course organizing concepts.

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